The meaning of logical connectives

we developed a calculus of reasoning which could verify that sequents of the form $\varphi 1, \varphi 2, ..., \varphi n \quad \psi$ are valid, which means: from the premises $\varphi 1, \varphi 2, ..., \varphi n$, we may conclude ψ. In this section we give another account of this relationship between the premises $\varphi 1, \varphi 2, ..., \varphi n$ and the conclusion ψ. To contrast with the sequent above, we define a new relationship, written

$$\varphi 1, \varphi 2, ..., \varphi n \quad \psi.$$

This account is based on looking at the ‘truth values’ of the atomic formulas in the premises and the conclusion; and at how the logical connectives manipulate these truth values. What is the truth value of a declarative sentence, like sentence (3) ‘Every even natural number > 2 is the sum of two prime numbers’? Well, declarative sentences express a fact about the real world, the physical world we live in, or more abstract ones such as computer models, or our thoughts and feelings. Such factual statements either match reality (they are true), or they don’t (they are false).

If we combine declarative sentences p and q with a logical connective, say $\wedge$, then the truth value of $p \wedge q$ is determined by three things: the truth value of p, the truth value of q and the meaning of $\wedge$. The meaning of $\wedge$ is captured by the observation that $p \wedge q$ is true iff p and q are both true; otherwise $p \wedge q$ is false. Thus, as far as $\wedge$ is concerned, it needs only to know whether p and q are true, it does not need to know what p and q are actually saying about the world out there. This is also the case for all the other logical connectives and is the reason why we can compute the truth value of a formula just by knowing the truth values of the atomic propositions occurring in it.

The map which assigns T to q and F to p is a valuation for $p \vee \neg q$. Please list the remaining three valuations for this formula. We can think of the meaning of $\wedge$ as a function of two arguments; each argument is a truth value and the result is again such a truth value. We specify this function in a table, called the truth table for conjunction. In the first column, labelled φ, we list all possible

φ	ψ	$\varphi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

The truth table for conjunction, the logical connective $\wedge$.